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The effect of the asymptotic response dynamics on the generalized synchronization

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Abstract. Generalized synchronization in a drive-response Chua circuits is investigated. A cascade of transitions to GS is observed with increasing the interaction strength. The mechanism on the transitions to GS is given based on the asymptotic behaviors of response dynamics.

PACS. 05.45.-a Nonlinear dynamics and chaos – 05.45.Xt Synchronization; coupled oscillators

Synchronization in interacting chaotic systems has been an active research field during the past decade because of its importance in nonlinear dynamics and potential applications in communication [1–4]. Chaos synchronization has been categorized as complete synchronization (CS) [5], generalized synchronization(GS) [6,7], lag synchronization (LS) [8] and phase synchronization (PS) [9]. Among these various types of synchronies, CS in interacting identical systems has been exhaustively investigated. As an extension of CS to nonidentical systems, GS implies the hooking of the output of one system to a given function of the output of the other system. Generally, GS is discussed in the master-slave configuration and has been found, numerically and experimentally, in the drive-response systems, the coupling systems and spatiotemporal systems [10–14]. Up to now, most of works focus on the search of GS in different systems and the development of methods on detecting GS. However a fundamental problem about the influences of asymptotic dynamical behaviors in master and slave dynamics on GS is seldom paid attention on. Furthermore there is an ordinary view that GS will be hold once it is established when the interaction between drive and response is stronger than a threshold value. However in this paper we will show that the relation between the status of GS and the strength of interaction is not simple: with the increase of the interaction strength, GS could be established, destroyed and reestablished. Based on the asymptotic behavior of response dynamics, an explanation on such a scenario is given.

The model adopted here is a drive-response system, where both the drive and response systems are Chua circuits. The system is written as

$$\begin{aligned}
\dot{x}_d &= \alpha[r_d(y_d - x_d) - f(x_d)] \\
\dot{y}_d &= r_d(x_d - y_d) + z_d \\
\dot{z}_d &= -\beta x_d \\
\dot{x}_{r,a} &= \alpha[r_r(y_{r,a} - x_{r,a}) - f(x_{r,a})] \\
\dot{y}_{r,a} &= r_r(x_{r,a} - y_{r,a}) + z_{r,a} + \varepsilon(y_d - y_{r,a}) \\
\dot{z}_{r,a} &= -\beta x_{r,a} \\
f(x) &= m_0 x + 0.5m_1[|x+a| - |x-a|] \\
&\quad + 0.5m_2[|x+b| - |x-b|]
\end{aligned} \tag{1}$$

where the subscripts d, r and a denote the coordinates for the drive, the response and the auxiliary systems (the second identical response system), respectively. ε is the interaction strength, parameters $\alpha = 10$, $\beta = 5.97$, $m_0 = 2.05$, $m_1 = -0.35$, $m_2 = -2.45$, $a = 2$, $b = 10.2$ are fixed. The bifurcation diagram for isolated Chua circuit in the range of $0.52 < r_d < 0.537$ is plotted in Figure 1 where the transitions from periodicity to chaos are observed. When $r_d < 0.534$, the Chua circuit has a single-scroll attractor (either periodic or chaotic dynamics); otherwise, double-scroll attractor is found.

In numerical simulations, the realization of GS can be detected by the auxiliary-system method or the negativeness of the largest conditional Lyapunov exponent (LCLE) in response dynamics [7]. For example in the auxiliary system method, GS is stable if we have $x_r = x_a$, $y_r = y_a$, $z_r = z_a$ no matter what the initial conditions for response and its auxiliary systems are.

First we let $r_d = 0.525$ and $r_r = 0.535$, which leads to that the drive system has a single-scroll chaotic attractor while the isolated response system has a double-scroll attractor. The status of GS is studied by varying the interaction strength ε. With the increase of ε, the alternation

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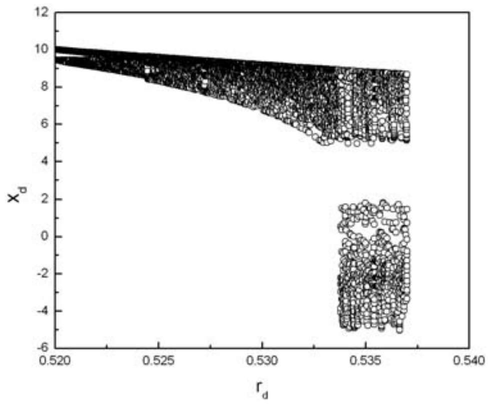

Fig. 1. The bifurcation diagram of isolated Chua circuit against r_d. The other parameters have been declared in the text.

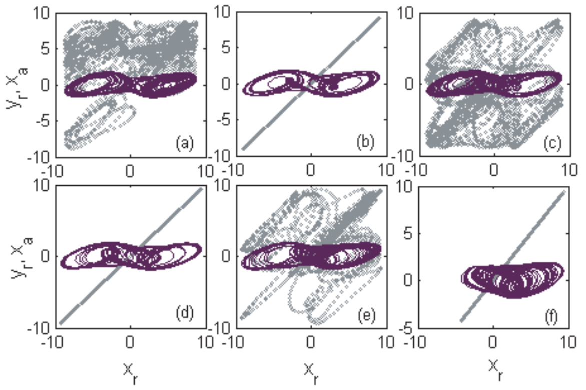

Fig. 2. The phase diagrams of the response system (black lines) and the relation between the response and auxiliary systems (gray lines) for the different coupling strength ε. $r_d = 0.525$, $r_r = 0.535$. (a–f) $\varepsilon = 0.01$, 0.59, 0.83, 2.0, 3.4, 5.0, respectively.

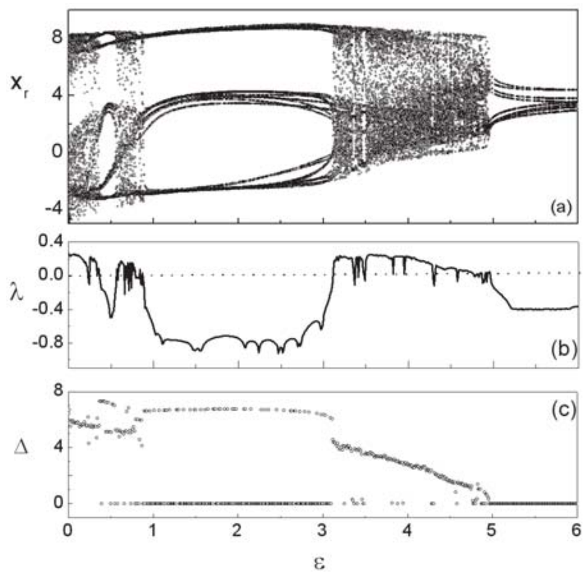

Fig. 3. (a) The bifurcation diagram of the response system. (b) The largest conditional Lyapunov exponent of the response system. (c) Synchronization error between the response and the auxiliary systems against the interaction strength. $r_r = 0.535$, $r_d = 0.521$.

between states of generalized synchronization and desynchronization can be found in Figure 2. Figure 2 exhibits the trajectory of response system on the plane of x_r and y_r (bold lines) and the relation between the response and its auxiliary system on the plane of x_r–x_a (gray circles) for different interaction strengths. It follows from Figure 2 that the status of GS between the drive and response systems remarkably relies on the interaction strength. In Figure 2a, the response system keeps its chaotic double-scroll attractor when the interaction is weak, i.e., $\varepsilon = 0.01$, and the relation between $x_r(t)$ and $x_a(t)$ is disordered. Clearly GS is not built yet. As the interaction strength increases to $\varepsilon = 0.59$, one can see that the attractor of the response system in Figure 2b becomes much more regular comparing with that in Figure 2a. Especially, the response and its auxiliary systems obey $x_r(t) = x_a(t)$ which means that GS happens. Interestingly, further increasing the interaction strength, for example $\varepsilon = 0.83$ in Figure 2c, GS is destroyed and the attractor of the response system becomes more irregular again. The status of GS is restored in Figure 2d where $\varepsilon = 2.0$ and the typical trajectory of the response system only shows weak irregularity. The alternation of GS and desynchronization can be repeated once more. For example, desynchronization (or GS) is observed in Figure 2e [or 2f] with $\varepsilon = 3.4$ (or $\varepsilon = 5$). It is worth to note that the GS in Figure 2f is different from those in Figures 2b and 2d in two respects: the typical trajectory in Figure 2f shows strong irregularity and GS will be hold even we further increase ε.

From Figure 2 we can know that the status of GS between the driving and response Chua circuits is strongly dependent on the interaction strength. Why does the transition to GS in equation (1) own such a scenario illustrated in Figure 2? Why do the typical trajectories of the response in Figures 2b and 2d look like regular orbits? To answer these questions, we consider the effects of different drive dynamics on the status of GS. We let $r_d = 0.521$, this makes the drive system to be periodic. To systematically investigate the GS between the drive and response systems, we first plot the bifurcation diagram of the response system against ε in Figure 3a. The bifurcation diagram displays the alternations of periodic and chaotic behaviors. Especially, double-scroll attractors for both periodic and chaotic response dynamics yield to single-scroll one at $\varepsilon < 4.97$. Furthermore, the LCLE of the response system is numerically computed and plotted in Figure 3b. Apart from narrow periodic windows with negative Lyapunov exponent, we can find in Figure 3b that there exist three large regimes with negative LCLE, i.e., $0.37 \le \varepsilon \le 0.58$, $0.79 \le \varepsilon \le 3.1$, and $\varepsilon \ge 4.97$. Considering the fact that the periodic regimes appeared in Figure 3 is forced by a periodic drive, it is clear that the regimes with negative LCLE correspond to those regimes with periodic behaviors. Since the negativeness of LCLEs means that GS between the drive and response systems has been

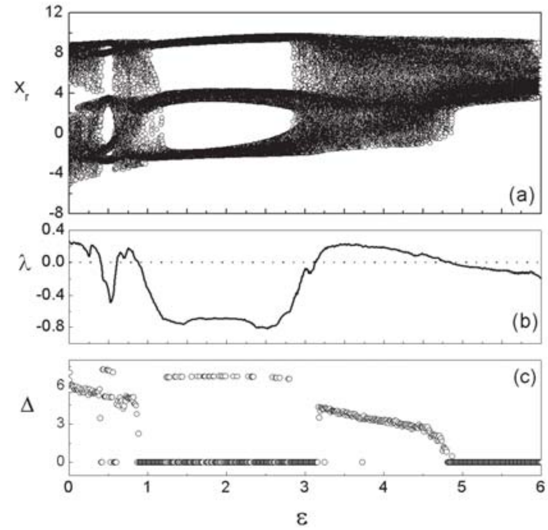

Fig. 4. (a–c) All curves are same as Figures 3a–3c with $r_d = 0.523$.

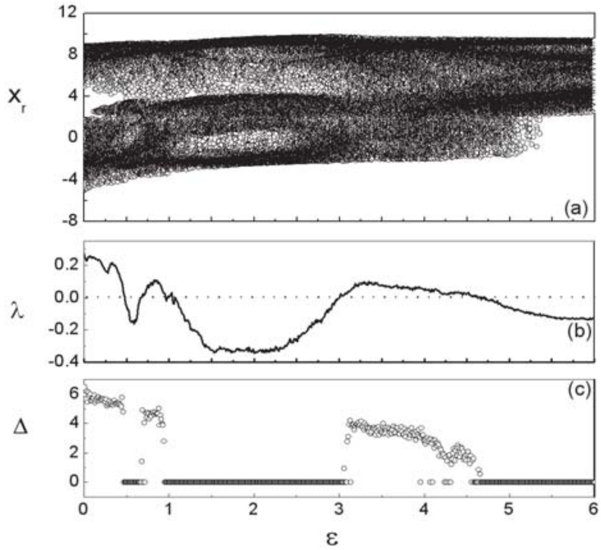

Fig. 5. (a–c) The same as Figure 3 with $r_d = 0.525$.

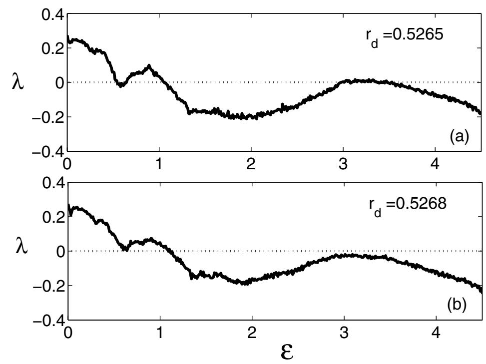

Fig. 6. (a, b) The largest conditional lyapunov exponent of the response system with $r_d = 0.5265$ and $r_d = 0.5268$, respectively.

established, we know that there is a direct correspondence between the status of GS and the periodicity in response dynamics when drive dynamics is periodic.

To further check the status of GS, we compute the synchronization error between the response and the auxiliary systems. The synchronization error is defined as

$$\Delta = \sqrt{\frac{1}{T}\sum\left[(x_r - x_a)^2 + (y_r - y_a)^2 + (z_r - z_a)^2\right]}. \quad (2)$$

Figure 3c shows the relation between Δ and ε. In contrast to the three large regimes with negative LCLE in Figure 3b, we can find in Figure 3c that the synchronization error Δ is equal to zero only for $\varepsilon \geq 4.97$ while Δ may be zero or nonzero in other two large regimes regardless of negative LCLE. Actually such a phenomenon comes from a pair of anti-phase solutions of the response system. This phenomenon, the drive system leads the response to different solutions even if GS is built, is not a coincidence and can be found in other response dynamics with symmetrical double-scroll structure, i.e., Lorenz system [15].

In Figure 3 we can see alternation of synchronization and desynchronization. The discussions above are for periodic drive dynamics, to investigate how the drive dynamics influences the transitions to GS and then how the results in Figure 3 are related to Figure 2, we let $r_d = 0.523$ where the drive system is in weak chaos (see Fig. 1). The bifurcation diagram against ε is shown in Figure 4a. It is interesting to find that Figure 4a is much similar to Figure 3a despite that the branches of periodic solutions in Figure 3a are broadened where irregularity steps in. The similarity between Figures 3a and 4a can be further confirmed by the plot of LCLE in Figure 4b and the synchronization error in Figure 4c. The three large regimes with stable GS locates at the roughly same places as Figure 3b regardless that there are no more periodic solutions. A heuristic explanation can be given on the fact that the stable GS regime is hardly influenced by drive dynamics. Though there is no periodic orbit in the GS regimes for weak chaotic drive, the broadened orbits will spend most of time in the neighbor of the previous periodic orbits under the periodic drive. Since the calculation of LCLE only involves the location of an orbit in the response phase space, we know the LCLE of the broadened orbit is most likely to be negative either by the consideration of continuity. Such an explanation can be extended to the drive dynamics deep into chaos, i.e., the drive system with $r_d = 0.525$. The bifurcation diagram in Figure 5a for the response system has almost lost the resemblance with Figure 3a. Nevertheless, the plots of the LCLE (see Fig. 5b) and the synchronization error (see Fig. 5c) still show the three regimes for GS as periodic drive does, which is also in concord with the results in Figure 2.

As increasing the parameter r_d of the drive, i.e., the drive system is more stronger chaotic, we can find a trend in Figure 6 that the first stable GS regime is disappearing, and the other two GS regimes is emerging when the drive parameter r_d is changing from 0.5265 (Fig. 6a) to 0.5268 (Fig. 6b). The phenomenon of transition to GS is asymptotically disappearing. To show this trend

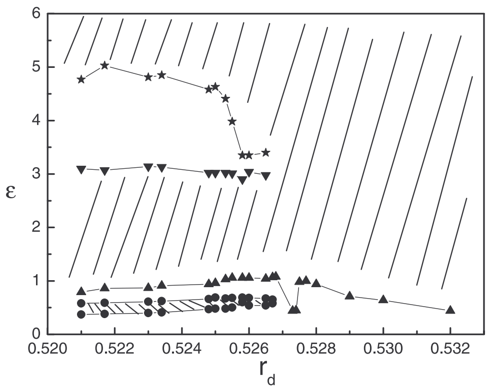

Fig. 7. The phase diagram for GS on the parameters r_d–ε plane. $r_r = 0.535$. The values of ε between two dot lines, two triangle lines, and above star line, correspond to regimes for GS. Lines and blank correspond to synchronization and desynchronization regimes.

completely, we plot the phase diagram for GS on the parameter plane of r_d and ε in Figure 7. In Figure 7 there are three stable GS regimes (corresponding to lines) when r_d arranges from 0.521 to 0.525, which correspond to change of drive system from periodic to weak chaotic state. This changing behaviors of the drive result in the asymptotic behaviors of the response (from Figs. 3 to 5). Continue to increase the drive parameter r_d to 0.527 in Figure 7, that means the drive is stronger chaos, three GS regimes have great changes, the first regime trends to shrink and the other regimes trend to emerge. This asymptotic behaviors of the response have great change while the parameter r_d is far away from 0.521. And when the drive dynamics is deep into chaos, i.e., $r_d > 0.5275$, there is only one GS regime for $\varepsilon > 0.98$ (we eliminate periodic windows near $r_d = 0.527$ and corresponding weak chaos regime), and this asymptotic effect of the response under the periodic drive completely disappears. In Figure 7 when $r_d > 0.5275$, we can observe the familiar GS phenomenon that the response and the drive systems reach GS and keep on when the coupling strength is larger than the threshold value. And the threshold value of the coupling becomes stronger as the parameter mismatch of the drive and the response is large. It is clear that the three large GS domains originated from periodic windows in previous figures do disappear and merge together with increasing r_d.

From above analysis, we can see that this transition of GS exists when the drive-response systems are satisfied some conditions, i.e., the drive is nearby the periodic orbits and shows weak chaos, and the response system by the periodic drive must have different GS regions. Actually we find similar phenomenon in the Rossler system and approve our analysis.

In summary, we have studied the GS in interacted Chua circuits. We find that there are several GS regimes separated by desynchronization regimes with the increase of interaction strength. By studying the influences of the drive dynamics on the status of GS, we find that the periodic windows in the response dynamics under the influence of the periodic drive play a key role on the formation of GS regimes. This phenomenon results from the asymptotic effect of response system under the weak chaotic dynamics of the drive system and it isn't restricted to the Chua system.

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